tion integral. The solution technique is based on defining an augmented matrix. A numerical example is provided which demonstrates the basic method for computing first- and second-order partial derivatives.

References

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Low-Thrust Insertion into Orbit around Mars

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Introduction

THE Mars Observer spacecraft is targeted for a circular mapping orbit of about 350-km altitude above Mars (radius = 3747.2 km) with an inclination of 98.8 deg and a 2 a.m. ascending node (sun synchronous). This orbit is achieved in two steps: 1) capture in an elliptical orbit around Mars and 2) subsequent circularization at the mapping orbit altitude. The initial elliptical orbit is also the phasing orbit for obtaining the correct ascending node position.

Theoretically, insertion of a spacecraft from its approach trajectory into the elliptical capture orbit around Mars can be accomplished by an impulsive burn at the common periapsis of the approach hyperbola and the capture ellipse. However, the Mars Observer spacecraft requires partial deployment of its appendages and solar array in the interplanetary cruise mode. The limited structural strength of these appendages requires restowage for high-thrust maneuvers. Rather than provide for restowing with attendant weight and reliability penalties, low-thrust insertion is proposed using four 110-lbf bipropellant thrusters, providing acceleration levels less than 0.15 g during the insertion burn.

Selection of the Elliptical Capture (Phasing) Orbit

For a launch between Aug. 20 and Sept. 9, 1990, and an arrival between Aug. 12 and Aug. 30, 1991, the capture orbit around Mars will have an ascending node at approximately 4 a.m.[1] A 30-deg node shift is then necessary in the phasing orbit to attain the required 2 a.m. node. Thereafter, a sequence of maneuvers is required to attain the mapping orbit. These maneuvers must be completed before the solar conjunction period between Oct. 24 and Nov. 24, 1991. Therefore, the primary consideration in choosing the phasing orbit is the attainment of the correct nodal rate.

For an impulsive burn, the target phasing orbit could be a circular orbit of 350-km altitude, but with an inclination of 90 deg. After achieving the correct nodal position, a subsequent burn could be effected to change the inclination to the required 98.8 deg.

With the 440-lbf thrust available on the Mars Observer spacecraft, capture into this 350-km phasing orbit requires a burn on the order of 30 min. This would contribute a finite-burn penalty of over 100 m/s because of the long arc. This penalty is the additional velocity increment required over an impulsive burn, resulting from the low thrust level that is accompanied by a relatively long thrust arc. To minimize this penalty, the plan is to inject into an elliptical capture orbit that, even with the same inclination as the final mapping orbit, provides the desired nodal shift. Table 1 shows the chosen nominal capture orbit which, for a launch on the first day of the window, accomplishes the required node shift in about 65 days. The target periapsis radius of this capture orbit is higher than the nominal 350 km to assure that the spacecraft stays above 300 km with 99% confidence in the presence of the navigational error (Table 2).[1] The northern approach to Mars has an advantage over the southern approach in that the resulting orbit has an ascending node 4 deg closer to that of the mapping orbit.

Approach Hyperbola and Burn Initiation

After the final trajectory correction maneuver (TCM 4 nominally 10 days prior to arrival at Mars), the spacecraft is targeted to the aim point (point B in Fig. 1) in the B-plane on the incoming leg of a hyperbolic trajectory. Because of the low thrust level (440 lbf), the insertion burn is estimated to last about 15 min, covering an arc of about 40 deg (Fig. 2). The figure also shows that during the burn, the periapsis of the orbit decreases. A burn over an arc of 40 deg results in a decrease of

Table 1 Nominal capture orbit for launch day 1

Parameter	Value
Semimajor axis	20,000 km
Periapsis radius	4,100 km
Eccentricity	0.795
Inclination	92.78 deg
Argument of perigee	92.63 deg

Table 2 Approach navigation uncertainty

Parameter	3-sigma value
Semimajor axis	330 km
Semiminor axis	180 km
Orientation angle	60 deg
Linearized time of flight	120 s

Table 3 Reduction in periapsis during insertion burn (using all four thrusters)

Percent of burn completed	Periapsis radius (km)
0	4150
25	4135
50	4128
75	4120
100	4100

Presented as Paper 87-0093 at the AIAA 25th Aerospace Sciences Meeting, Reno, NV, Jan. 12–15, 1987; received Feb. 2, 1987; revision received May 22, 1987.

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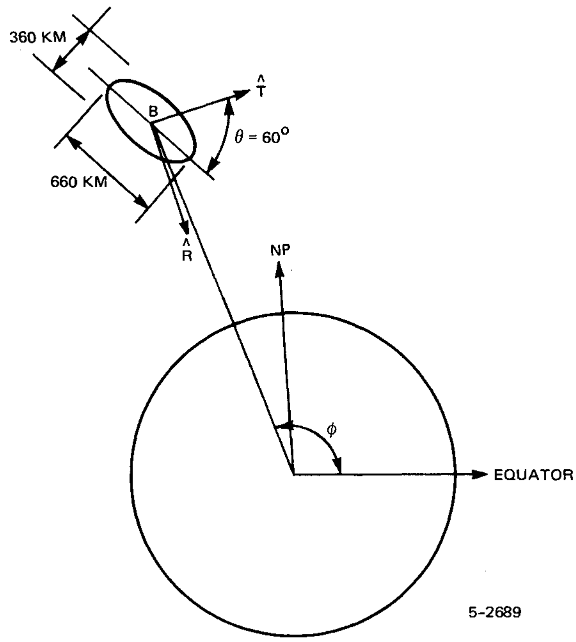

Fig. 1 Location of aim point (B) and 3σ navigation error ellipse.

50 km in the periapsis altitude. Table 3 shows the periapsis radius as a function of the completed burn. Therefore, at TCM 4 the target periapsis of the approach hyperbola is taken at 4150 km, so that by the end of the burn, it would reduce to the desired 4100-km radius.

The initiation point of the burn (θ_s in Fig. 2) determines the periapsis radius discussed and the finite-burn penalty. To minimize these two effects, the burn is initiated 20 deg prior to periapsis (θ_s is equal to θ_e in Fig. 2), thus effectively centering the burn around the periapsis. Burn simulations using various pitch profiles showed that the finite-burn penalty is minimized by selecting a steering law such that the thrust vector is along the negative velocity vector. This profile produces the most efficient change in spacecraft velocity. By optimally choosing the initiation point and the steering law as discussed, the finite-burn penalty to achieve the 4100 × 20,000 km capture orbit can be kept to below 20 m/s (over an impulsive requirement of 985 m/s).

Navigation and Attitude Determination

The Mars Observer spacecraft will be tracked by the Deep Space Network (DSN) during the interplanetary cruise orbit and the phasing orbit. The navigation and tracking data link is maintained by the spacecraft's high-gain antenna (HGA). Orbit determination will be performed on the ground using the HGA ranging and Doppler data; ephemerides will be generated on board the spacecraft by an ephemeris predictor that integrates accelerometer output through the burn. The ephemerides will provide spacecraft position and velocity vectors. In addition, the spacecraft will have onboard planetary ephemerides containing information on sun, Earth, and Mars, thus providing a reference for the spacecraft's position and orientation.

During the interplanetary and phasing orbits, the spacecraft's inertial attitude will be determined by integrating gyro-sensed body rates with periodic updates (every few hours) from a star mapper. The inertial attitude, in conjunction with the spacecraft and planetary ephemerides (Earth and sun), provides a reference for the HGA to be pointed toward the Earth and for controlling the spacecraft's coning about the sun line. During the insertion burn, the spacecraft body rate is nulled and the attitude is slewed to the desired orientation using reaction wheels. During the burn, a programmed pitch rate keeps the thrust tangential to the spacecraft velocity. After the maneuver is completed, the spacecraft is slewed back to the sun-spinning configuration. During this entire period, spacecraft attitude is determined exclusively by gyros. Star updating is resumed after the sun-spinning mode has been re-established.

The insertion burn is accomplished with four 110-lbf bipropellant thrusters, which are offpulsed for two-axis control purposes. Control about the thrust axis is achieved by onpulsing a separate set of thrusters.

Insertion into Phasing Orbit Using Backup Half-System

A stipulation for backup half-system insertion capability requires the capture burn to be executed using only two of the four 110-lbf bipropellant thrusters, if necessary. This will reduce the thrust level by a factor of two and substantially increase the thrust arc. To reduce the accompanying finite-burn penalty and to avoid getting an unacceptably low periapsis, capture will be executed in two burns. The first burn will capture the spacecraft into a highly elliptical orbit around Mars, and a second burn at a subsequent periapsis will take the spacecraft to the desired elliptical drift orbit. For half-system opera-

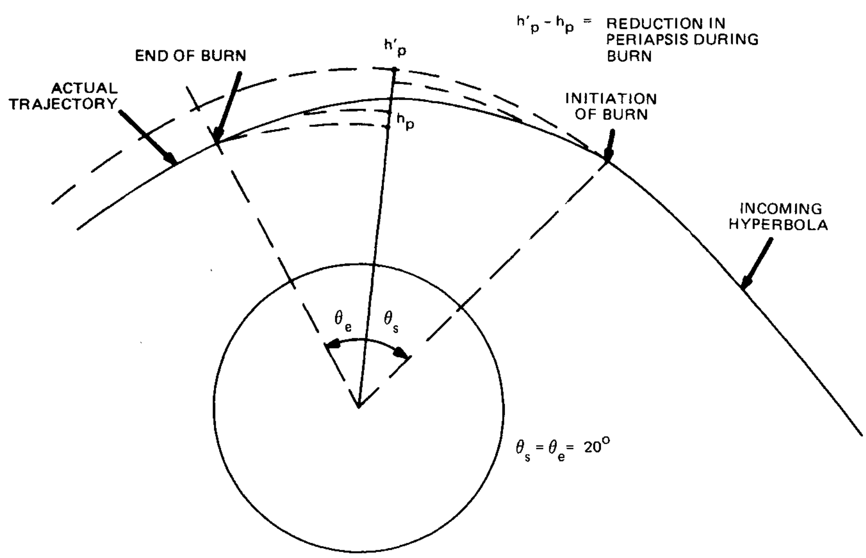

Fig. 2 Mars orbit capture scenario (using all four thrusters).

Table 4 Capture orbit initiation for day 1 of launch window (full- and half-system scenarios)

	Desired orbit						Achieved orbit		
Mode	Periapsis radius (km)	Semimajor axis (km)	Impulsive velocity requirement (m/s)	Approach periapsis (km)	Burn initiation point θ_s (deg)	Burn time (min)	Periapsis radius (km)	Semimajor axis (km)	Actual velocity increment (m/s)
Full system[a]	4100	20,000	985	4160	40	14	4100	20,000	1000
Half system[b]	4100	20,000	985	4160	40	25 3[c]	4105 4100[c]	40,000 20,000[c]	900 100[c]

[a]Full system uses all four thrusters. [b]Half system uses two of the four thrusters. Capture executed in two burns. [c]Second burn.

tion, if the burn initiation point were to be the same as for the full system (θ_s is 20 deg in Fig. 3) and the same steering law were to be utilized (thrust in the negative velocity direction), the periapsis radius would become smaller than the radius of Mars before the spacecraft velocity could be sufficiently reduced to capture the spacecraft around Mars. Therefore, for backup half-system operation, the required burn initiation position is 40 deg before the periapsis (θ_s is 40 deg in Fig. 4).

Because the requirement for half-system operation can be determined only after initiation of the burn, the burn initiation point for the full system is also retracted to 40 deg before periapsis to assure viable half-system capability. The full-system burn will thus no longer be centered around the periapsis; this imposes an additional 5 m/s in finite-burn penalty and reduces the periapsis radius by an additional 10 km. This requires a target periapsis radius of 4160 km at TCM 4 to obtain the desired 4100 km at the end of the insertion burn. Table 4 shows the capture scenario for a redundant system (for full- or half-system operation).

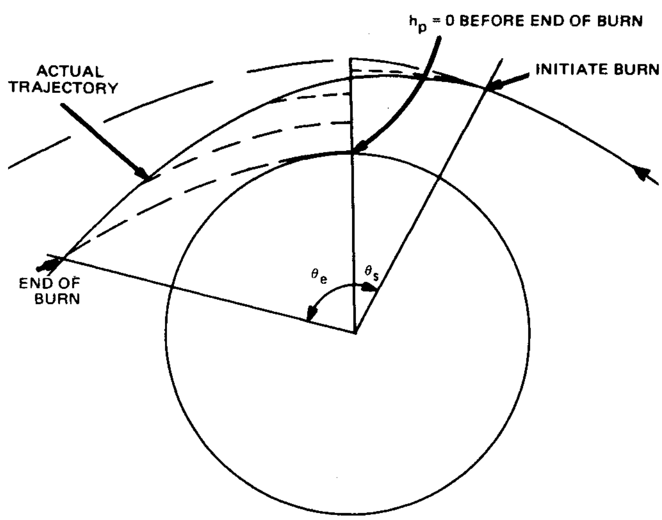

Fig. 3 Capture using two thrusters and $\theta_s = 20$ deg.

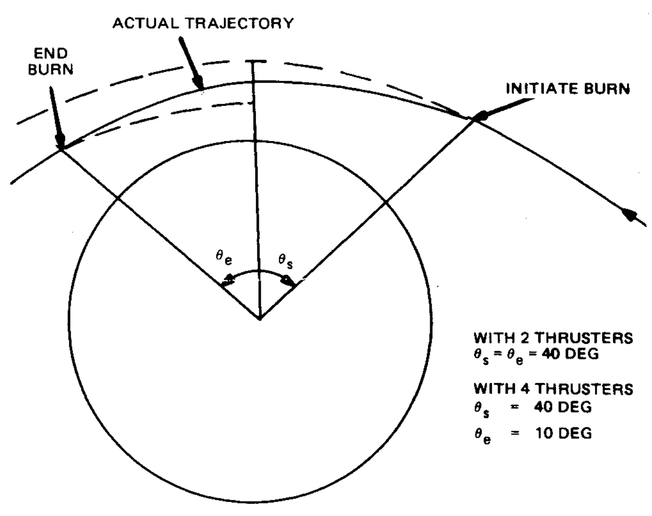

Fig. 4 Capture using two or four thrusters and $\theta_s = 40$ deg.

Conclusion

This paper has discussed the implication of a low-thrust insertion into a Mars orbit using four 110-lbf thrusters. The strategy includes insertion into an elliptical capture (drift) orbit (4100 × 20,000 km), targeting for a 4150-km periapsis to compensate for reduction in periapsis altitude during burn, initiating the burn at about 20 deg before the periapsis (40 deg if half-system capability is desired), and minimizing the finite-burn penalty by pitching the spacecraft during the burn to align the thrust vector parallel to the negative velocity vector.

Reference

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Tethered Subsatellite Swinging from Atmospheric Gradients

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Nomenclature

C_D = aerodynamic drag coefficient
D = aerodynamic drag force
EA = longitudinal rigidity of tether
l = tether length
l_o = tether length in zero-tension state
m = mass of subsatellite
R = radius of orbit of main satellite
S = equivalent drag area (involving both subsatellite and tether)
T = tether tension

Received March 27, 1987; revision received June 4, 1987.

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